Fermion Masses and Mixings
in a String-Inspired Model

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Abstract

In this paper we study quark and lepton mass matrix textures in a model containing an additional $U(1)_X$ gauge symmetry with origins in string compactification. The $U(1)_X$ symmetry is broken near the string scale, and we assume that the anomalies are canceled by the Green-Schwarz mechanism. We also assume that fermion mass matrices are generated by an additional scalar field through an approach analogous to that of Froggatt and Nielsen. By requiring that supersymmetry not be broken at the high scale, we can derive the vacuum expectation value of this scalar field to then predict fermion masses and mixings for any given X charge assignment. We examine the possible solutions, and although in the simplest model they do not completely agree with experiment, the results are close enough to merit further inspection.

1 Introduction

Understanding the fermion mass structure has been a goal of particle theorists for some time. In 1978, Froggatt and Nielsen [1] found that a spontaneously broken family dependent symmetry could naturally explain the large mass ratios among different families of quarks and leptons. Renormalizing experimental data to the Planck scale reveals the order of magnitude estimates to the following ratios [2, 3, 4]:

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda^8) \; ; \qquad \frac{m_d}{m_b} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4)$$

$$\frac{m_c}{m_t} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_s}{m_b} = \mathcal{O}(\lambda^2) \; ; \qquad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda^2) \; , \qquad (1)$$

where $\lambda \simeq 0.22$ is the small parameter used in the Wolfenstein's parametrization [5] of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [6, 7].

In the past few years, there has been a revival of theories which predict a mass hierarchy from a spontaneously broken family symmetry. This time the work has been done in the context of supersymmetry (SUSY). The general idea has been used widely in more detailed models with family symmetries that were continuous and discrete, Abelian and non-Abelian, global and local, and with different choices for the symmetry breaking scale [4, 8, 9, 10, 11, 12].

One of the unanswered questions of the original Froggatt-Nielsen model is the origin of the family symmetry breaking. It has been suggested [4, 11] that the supersymmetric versions of the model may be derived from superstring compactification, where spontaneously broken anomalous $U(1)$ gauge symmetries typically occur. In models where the anomalies are canceled by the Green-Schwarz mechanism [13], the symmetry breaking scale is slightly below the string scale. Preserving supersymmetry at the high scale determines the vacuum expectation value (VEV) of the symmetry breaking field θ and the hierarchy parameter λ, which greatly restricts the theory. The purpose of our work is to find out whether this new constraint can be accommodated in a phenomenologically acceptable model.

In this paper, we present models which predict the fermion masses and mixings in a string-inspired framework. Since we do not work with an exact string model, we carry a model-independent analysis as far as possible. In doing so we make the following assumptions: (1) the additional Froggatt-Nielsen symmetry is an anomalous $U(1)$ originating in string theory so that

the anomalies are canceled by the Green-Schwarz mechanism; (2) renormalization of couplings and particle masses is done within the framework of the minimal supersymmetric standard model (MSSM); (3) the $U(1)$ symmetry is broken by the VEV of only one field, θ; (4) the Yukawa coupling of θ to the fermions f_θ is one. For most of the paper, we assume that the Kac-Moody level, k_{GUT}, for the Grand Unified Theory (GUT) group is one, but we keep k_{GUT} as a parameter in most equations. Finally, in the context of a particular string model, the $U(1)$ symmetry breaking field(s) will be known, as will the value of f_θ and k_{GUT}.

The outline of the paper is as follows. In section 2, we summarize the main features of the Froggatt-Nielsen models. In section 3 we discuss the implications of unbroken supersymmetry for the value of the mass hierarchy parameter, λ. This is followed by section 4, which gives the background and some important facts about the Green-Schwarz mechanism. Section 5 is dedicated to the anomalies of the model. We show there how λ depends only on the X charges of the standard model fields. In section 6 we present some constraints on those charges following from relations (1). We illustrate these constraints in section 7 by working out in detail a few simple models. Section 8 summarizes our results.

2 Froggatt-Nielsen Models

Originally, Froggatt and Nielsen proposed [1] that a flavor-dependent symmetry be broken by the VEV of an additional scalar field, θ, which would be a singlet of the standard model gauge groups. Their idea also assumed a set of heavy "mirror quarks", analogous to the standard model quarks, with a spectrum of charges under the horizontal symmetry. Mass matrices would then arise through effective Yukawa interactions resulting from Feynman diagrams such as that in figure 1.

In figure 1, we show an example, where the X charge assignments of the quarks are written above each quark line. On one side of the diagram, we have a left-handed quark doublet with charge $+2$. If we assign a charge of -3 to the right-handed quark, then there must be five θ interactions, with five mirror quarks of charges $+2$, $+1$, 0, -1, -2 in between. The first mirror quark of charge $+2$ interacts with the standard model Higgs doublet and the quark doublet to conserve $SU(2)$ symmetry. Assuming a common mass M
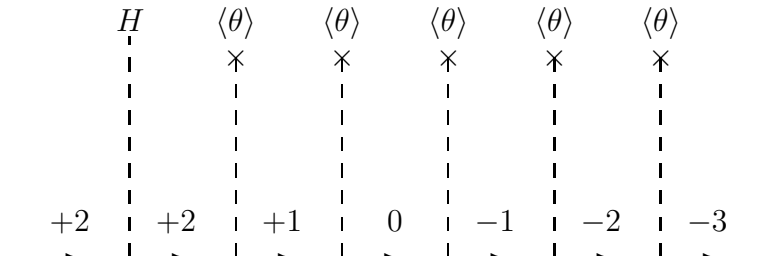
Figure 1: Tree diagram leading to effective Yukawa interactions. Above the solid line are $U(1)$ charges of quarks and mirror quarks.

for the mirror quarks, and a common Yukawa coupling f_θ of the θ field to all the quarks, $\langle\theta\rangle$ should take a value such that $\lambda \sim f_\theta\langle\theta\rangle/M$. The mass term resulting from fig. 1 would be

$$ f_u \bar{u}_j Q_i H \left(f_\theta \frac{\langle\theta\rangle}{M} \right)^5 . \tag{2} $$

In a realistic model, the charges must be assigned so that all the mass matrix eigenvalues agree with the relations (1) above.

Here, we do not make the Froggatt-Nielsen assumptions, and thus do not require mirror quarks. We assume that the flavor symmetry is a gauged $U(1)$ symmetry, labeled by X, left over from string compactification. We expect the action to contain all terms consistent with charge conservation. Such terms appear due to string tree diagrams; therefore, the effective Yukawa coupling f_θ will be a product of the string coupling constant g_s and other terms of order unity. Not knowing the details of the model, we cannot evaluate f_θ, and here assume $f_\theta = 1$. In order to demonstrate the generation of mass terms, we give an explicit example. We do not make any further assumptions about the physical mechanism.

First, we define the X charges to be q_{Qi}, q_{ui}, q_{di}, q_{Li}, and q_{ei} for the left-handed quark doublets, the left-handed up-type antiquarks, the left-handed down-type antiquarks, the left-handed lepton doublets and the left-handed positrons (i is the family index). Also, we define q_H to be the sum of the X charges for the two Higgs doublets of the supersymmetric standard model.

We then consider all bilinear fermion terms that conserve X charge. A typical up-type quark term would be

$$(\mathbf{Y}_u)_{ij} = f_u \bar{u}_j Q_i H_1 \left(\frac{\langle \theta \rangle}{M} \right)^{q_{Qi}+q_{uj}+q_{H1}} \tag{3}$$

The entire Yukawa mass matrix then follows:

$$\mathbf{Y}_u = f_u \, \lambda^{q_{H1}} \begin{pmatrix} \lambda^{q_{Q1}+q_{u1}} & \lambda^{q_{Q1}+q_{u2}} & \lambda^{q_{Q1}+q_{u3}} \\ \lambda^{q_{Q2}+q_{u1}} & \lambda^{q_{Q2}+q_{u2}} & \lambda^{q_{Q2}+q_{u3}} \\ \lambda^{q_{Q3}+q_{u1}} & \lambda^{q_{Q3}+q_{u2}} & \lambda^{q_{Q3}+q_{u3}} \end{pmatrix}. \tag{4}$$

3 Implications of Unbroken Supersymmetry

The basic premise of this work is the assumption of a deeper connection between string theory and supersymmetric models with spontaneously broken family symmetries. By assuming such a connection, we can "borrow" a $U(1)$ gauge symmetry left over from string compactification.

We begin with $N = 1$ global supersymmetry and the scalar potential:

$$V = \frac{1}{2} \sum_\alpha (D^\alpha)^2 + \sum_i |F_i|^2. \tag{5}$$

Here $F_i = \partial W / \partial \phi_i$. We do not specify the superpotential W; thus, we will not be able to predict the VEV's of all of the fields in the model. The gauge D term is given by $D^\alpha = g_{(\alpha)} \sum_{ij} \phi_i^* (T^\alpha)_{ij} \phi_j$, where ϕ_i are the matter chiral superfields, T^α are the generators of the gauge group, and $g_{(\alpha)}$ are the gauge couplings. For an anomalous $U(1)$ gauge group, the corresponding D term will be modified by a Fayet-Iliopoulos term. Its magnitude has been calculated in string theory [14, 15, 16] on the assumption that the anomalies are canceled by the Green-Schwarz mechanism [13]:

$$D = \frac{g_s M_s^2}{192\pi^2} \operatorname{tr} Q + \sum_i q_i |\phi_i|^2, \tag{6}$$

where g_s is the renormalized string coupling constant, M_s is the string scale, and $\operatorname{tr} Q \equiv \sum_i q_i$ is the sum of the $U(1)$ charges of all the particles. (See also [17] for a clear exposition.) In the model we are studying, the $U(1)_X$

family symmetry gauge group is anomalous, has origins in string theory and requires a D term given by eq. (6).

The supersymmetric vacuum requires $\langle F_i \rangle = 0$ and $\langle D^\alpha \rangle = 0$. If the X charges of all the particles are of the same sign, then, according to eq. (6), it is impossible to preserve supersymmetry. We therefore require that charges with both signs be present. (Fortunately, this is typically the case in string compactifications.) As a convention, we give the θ field a negative charge. In section 7, we will consider both the case in which all the standard model matter fields have positive X charge and the case in which they can have charges of either sign, subject to the condition $\mathrm{tr}\, Q > 0$.

We have to assume that none of the fields charged under the standard model (SM) gauge groups can develop vacuum expectation values—otherwise color or electroweak symmetry would be broken at the high scale $\sim M_s$. The problem of flat directions here does not differ from the problem of flat directions in the MSSM. We require $\langle D^\alpha \rangle = 0$ for each gauge factor separately; for each of the SM gauge groups this condition involves only SM fields, so that the flat directions will be the same as in the MSSM. For $U(1)_X$, setting $\langle D \rangle = 0$ merely determines $\langle \theta \rangle$ and does not constrain MSSM fields. Hence, just as in the MSSM, we have to rely on the superpotential to lift the flat directions.

The most important implication of eq. (6) for this work is that preserving supersymmetry determines the VEV of the θ field

$$\frac{\langle \theta \rangle}{M_s} = \sqrt{\frac{g_s}{192\pi^2} \frac{\mathrm{tr}\, Q}{|q_\theta|}} \; . \tag{7}$$

If we assign X charges to all the fields and use θ as the family symmetry breaking field in the Froggatt-Nielsen scheme, we obtain a prediction for the Yukawa mass matrices.

The question we attempt to answer in this paper is: can we find a set of charges for all the standard model fields, consistent with the requirements of anomaly cancellation, that will predict phenomenologically viable powers of λ (much like the work by Ibáñez and Ross [11], but without the assumption of left-right symmetry) *and* predict a phenomenologically viable value of $\lambda \simeq 0.22$? We shall see that this is possible as long as f_θ is not very different from unity.

Our results, as can be seen from eq. (7), are rather sensitive to the value of the string coupling constant, g_s. At the unification scale, we use the tree-level

relation [18, 19]

$$1/g_s^2 = k_{GUT}/g_{GUT}^2, \tag{8}$$

where k_{GUT} is the Kac-Moody level for corresponding GUT gauge group algebra. Here, we take $k_{GUT} = 1$. We then use a typical value $\alpha_{GUT} = g_{GUT}^2/4\pi \simeq 1/25$ to get $g_s \simeq 0.7$. Substituting into eq. (7) with $q_\theta = -1$, we obtain

$$\lambda = \langle\theta\rangle/M_s = 1.92 \times 10^{-2} \sqrt{\operatorname{tr} Q}. \tag{9}$$

4 Green-Schwarz Anomaly Cancellation

In chiral theories, it is necessary to consider the problem of quantum anomalies. These anomalies to classical symmetries are dangerous in that they prevent the existence of gauge theories. In this section we discuss a method of removing anomalies that may arise with a new $U(1)_X$ gauge symmetry.

We start with $U(1)$ chiral transformations on all fermions:

$$\begin{aligned}
\Psi(x) &\rightarrow \exp\left[-iq\gamma_5\Theta(x)\right]\Psi(x) \\
\bar\Psi(x) &\rightarrow \bar\Psi(x)\exp\left[-iq\gamma_5\Theta(x)\right],
\end{aligned} \tag{10}$$

where q is the charge of each fermion. Since the path integral measure is not invariant under the transformation, we obtain new terms beyond the usual current divergence [20]. The difference can be expressed as a change in the Lagrangian:

$$\mathcal{L} \rightarrow \mathcal{L} - \Theta(x)\sum_{i=1,2,3,X}C_i F_i \tilde{F}_i - \Theta(x)\left(\partial_\mu j_5^\mu\right) \tag{11}$$

where the sum is over the standard model gauge groups and $U(1)_X$. The coefficient $C_1 = \operatorname{tr}[Q(Y/2)^2]$ is the mixed $U(1)_X\,(U(1)_Y)^2$ anomaly, $C_X = \operatorname{tr}Q^3$ is the $(U(1)_X)^3$ anomaly, and $C_{2,3} = \frac{1}{2}\operatorname{tr}_{2,3}Q$ are the $U(1)_X\,(SU(2)_L)^2$ and $U(1)_X\,(SU(3)_c)^2$ anomalies. (The trace $\operatorname{tr}_{2,3}Q$ is over fermions with $SU(2)_L$ and $SU(3)_c$ charge, respectively.) In some cases we can choose the charges so that all the anomaly coefficients are zero, but here we examine the possibility of canceling the anomalous term with another of opposite sign [13]. There is another mixed anomaly with $U(1)_Y$, the $(U(1)_X)^2\,U(1)_Y$, but it does not fit into the discussion. It results from a different transformation than that in eq. (10), where q would be the $U(1)_Y$ instead of the $U(1)_X$ charge.

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In the Green-Schwarz mechanism, anomalies are canceled through an additional field. In string theory, the antisymmetric tensor $B_{\mu\nu}$ naturally serves this purpose. In four dimensions [21], we can replace $H = dB$ with its dual, which is the derivative of the axion field Φ:

$$dB = *d\Phi. \tag{12}$$

This field couples to the gauge groups in the following way:

$$\Phi \sum_{i=1,2,3,X} k_i F_i \tilde{F}_i, \tag{13}$$

where k_i are the Kac-Moody levels of the corresponding gauge algebra. For the $U(1)_X$ gauge transformation,

$$A_X^\mu \to A_X^\mu + \partial^\mu \Theta(x) \tag{14}$$

Φ follows the transformation

$$\Phi \to \Phi + \Theta(x)\, \delta_{GS}. \tag{15}$$

Therefore, we can remove quantum anomalies through a gauge transformation if $\delta_{GS} = C_1/k_1 = C_2/k_2 = C_3/k_3 = C_X/k_X$. For a more detailed discussion, we refer the reader to the paper by Ibáñez [22].

So far, we have ignored gravity, but the conclusions do not change. We must only cancel one additional anomaly, $C_{\text{grav}} R\tilde{R}$, through a gauge transformation on one additional coupling $k_{\text{grav}} \Phi R\tilde{R}$. Finally, we have:

$$C_1/k_1 = C_2/k_2 = C_3/k_3 = C_X/k_X = C_{\text{grav}}/k_{\text{grav}}. \tag{16}$$

In this paper, we take $k_1 = 5/3$ and $k_2 = k_3 = k_{\text{grav}} = 1$, and we do not use k_X, so that

$$C_1 : C_2 : C_3 : C_{\text{grav}} = \frac{5}{3} : 1 : 1 : 1. \tag{17}$$

5 Quantum Anomalies

We can now apply the results of the previous section to the case at hand. The mixed anomalies with the standard model gauge groups are

$$C_1 = \frac{1}{6}(q_Q + 8q_u + 2q_d + 3q_L + 6q_e + 3q_H)$$

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$$C_2 = \frac{1}{2}(3q_Q + q_L + q_H) \tag{18}$$

$$C_3 = \frac{1}{2}(2q_Q + q_u + q_d),$$

where $q_Q = \sum_{i=1}^{3} q_{Qi}$, etc., but $q_H = q_{H1} + q_{H2}$ is the sum of the $U(1)_X$ charges of the two Higgs doublets. Our calculations do not include the right-handed neutrinos, but since we allow for the existence of additional particles with X charge that are singlets under the standard model, our results do not depend on the existence and X charge assignments of ν_R.

The gravitational anomaly is given by

$$C_{\text{grav}} = \frac{1}{24} \sum_{\text{all particles}} q_i = \frac{1}{24}(6q_Q + 3q_u + 3q_d + 2q_L + q_e + 2q_H + q_\theta + q_X), \tag{19}$$

where q_X is the sum of the $U(1)_X$ charges of any additional fields which are singlets under the standard model. We are not excluding such fields, and we cannot evaluate C_{grav} directly. However, because we are using the Green-Schwarz mechanism, we know that C_{grav} must be in the correct proportion (17) to the other anomalies. From the expressions (18) and (19) we then obtain

$$q_\theta + q_X = 18q_Q + 8q_u + 7q_d = 14C_3 + 4q_Q + q_u. \tag{20}$$

With the assumption that θ is the only field with a negative $U(1)_X$ charge, we see immediately that we *require* the extra fields with no standard model interactions to balance eq. (20) with a large positive contribution q_X. If we allow the quarks to have negative charges, this is no longer true. Even then, in section 6 we shall see that $C_3 \sim 9$ in phenomenologically interesting models, so that the typical model will require $q_X > 0$.

The additional fields responsible for $q_X \neq 0$ prevent us from calculating the cubic anomaly $(U(1)_X)^3$, which then does not impose any constraints on the model. We simply assume that the charges of the extra fields are such that it is canceled:

$$C_X = \sum_{\text{all particles}} q_i^3. \tag{21}$$

On the other hand, the mixed anomaly $(U(1)_X)^2 \, U(1)_Y$,

$$C_{YXX} = \sum_{\text{all particles}} Y_i q_i^2 = \sum_{i=1}^{3} \left(q_{Qi}^2 - 2q_{ui}^2 + q_{di}^2 - q_{Li}^2 + q_{ei}^2 \right) - q_{H1}^2 + q_{H2}^2, \tag{22}$$

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depends only on the charges of the standard model particles and cannot be canceled by the Green-Schwarz mechanism. For each charge assignment we have to check that $C_{YXX} = 0$.

The equality of C_3 and C_{grav}, eq. (17), is crucial in this paper. Section 3, eq. (7) gave us a prediction for the hierarchy parameter λ in terms of $\text{tr}\,Q$, the sum of the $U(1)_X$ charges of all the particles in the model. That sum, according to eq. (19), is proportional to the gravitational anomaly. Because $C_{\text{grav}} = C_3$, every charge assignment for the standard model fields (in fact, for the quark fields alone) results in a prediction

$$\lambda = \sqrt{g_s/8\pi^2}\,\sqrt{C_3} \simeq 0.094\sqrt{C_3}. \tag{23}$$

6 Determinants of the Mass Matrices

The product of the determinants of up- and down-quark mass matrices will give us an important constraint. From eq. (1), it is immediately seen that

$$\prod_{\text{all quarks}} m_q \sim f_u^3 f_d^3 \lambda^{18}. \tag{24}$$

This should be compared to the product of the determinants of the Yukawa matrices predicted by the model. Writing $\mathbf{Y}_u$ in the form (4), we see that every term in the determinant is of the order $f_u^3 \lambda^{q_Q + q_u + 3q_{H1}}$. Similarly, every term in $\det \mathbf{Y}_d$ is of the order $f_d^3 \lambda^{q_Q + q_d + 3q_{H2}}$. The two taken together give [23]

$$\prod_{\text{all quarks}} m_q = |\det \mathbf{Y}_u|\,|\det \mathbf{Y}_d| \sim f_u^3 f_d^3 \lambda^{2q_Q + q_u + q_d + 3q_H}. \tag{25}$$

Now, $q_H = 0$ if a μ term $\mu H_1 H_2$ is to be allowed and not suppressed. (For alternatives, see [23, 24]. We note that a small change in q_H can be easily accomodated, as it will not change the predicted mass ratios or mixings.) Then, using eq. (18), we are left with

$$\prod_{\text{all quarks}} m_q \sim f_u^3 f_d^3 \lambda^{2C_3}. \tag{26}$$

From equations (24) and (26), we see that

$$\lambda^{18} \sim \lambda^{2C_3}, \tag{27}$$

or

$$C_3 \simeq 9, \tag{28}$$

as was found earlier by Binétruy and Ramond [4]. This is true whether or not there are texture zeros, provided that neither determinant is zero. For $C_3 = 9$, eq. (7) gives $\lambda = 0.28$.

The above reasoning assumes that λ is fixed at about 0.22—the mass ratios (1) come from experiment, not from assumptions about hierarchy. In this paper, we derive the hierarchy parameter λ from supersymmetry. Taking λ as predicted by eq. (23), we have to replace (27) by

$$0.22^{18} \sim \left(0.094\sqrt{C_3}\right)^{2C_3}. \tag{29}$$

Solving this, we get $C_3 \simeq 12.5$ and $\lambda \simeq 0.33$, rather than 0.22.

This value of λ will restrict the number of solutions because first order calculations predict a dependency of the Cabibbo angle on λ. If Yukawa matrices are given in terms of powers of λ, so will, to leading order, the CKM matrix [1]. The experimental uncertainty on the average value of the Cabibbo angle [25, 26]

$$|\mathbf{V}_{12}| = 0.2205 \pm 0.0018 \tag{30}$$

is very small. Keeping in mind that, as noted by Olechowski and Pokorski [2], $|\mathbf{V}_{12}|$ is almost invariant (it changes by less than 0.1%) when renormalized from M_W to M_{GUT}, we will always try to keep close to $\lambda \simeq 0.22$.

In order to remedy the solution to eq. (29), we will examine the assumptions that play a significant role since the equation itself is robust. It is robust because C_3 is related to the exponent of a small parameter, so a small change in C_3 would change the determinants by orders of magnitude. With $\lambda \simeq 0.22$, we estimate that unless the order unity factors in the Yukawa matrices all conspire to shift the balance in one direction, they could increase C_3 by as much as two or three. One should also note that there are no top or bottom Yukawa couplings in eq. (27), so the result is independent of $\tan \beta$.

One assumption that affects the value of C_3 more significantly is $f_\theta = 1$. If, for example, f_θ were 0.78, then the expected and calculated values of λ would be reconciled.

Another assumption that can be relaxed concerns the values of the Kac-Moody levels. If, instead, $k_2 = k_3 = k_{GUT} = 2$ while $k_{\mathrm{grav}} = 1$, then both

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eq. (8) relating g_s and g_{GUT} and eq. (16) relating C_3 and C_{grav} must change. (Models with $k_{GUT} = 2$ have been increasingly popular with string theorists [27, 28, 29].) The final relation for λ, eq. (23), becomes

$$\lambda = f_\theta \sqrt{\frac{g_{GUT} \, C_3}{8\pi^2 \sqrt{k_{GUT}}}} \simeq 0.079 f_\theta \sqrt{C_3}. \tag{31}$$

Now, $\lambda \sim 0.24$ when $C_3 = 9$.

Lastly, we note that λ also depends on the value of g_s by the above equation (for both $k_{GUT} = 1$ and $k_{GUT} = 2$). However, we know that f_θ has a linear dependence on g_s, so that

$$\lambda \propto g_s^{3/2}. \tag{32}$$

In order to attain a value of $\lambda \sim 0.22$, g_s would have to be reduced from 0.7 to 0.59. This would require $\alpha_{GUT} = 1/36$, which is too low according to most models.

7   Detailed Examples

We are now ready to examine in detail the $U(1)_X$ charge assignments, which, subject to the constraints discussed in sections 5 and 6, let us calculate fermion mass matrices. We would then compare the quark and lepton masses with relations (1) and demand a phenomenologically viable CKM matrix. Ideally, among all the possible charge assignments we would find at least one that satisfies all the constraints and predicts masses and mixings within experimental bounds.

Although the fifteen charges of the quark and lepton fields may seem like many free parameters, they are in fact overconstrained. If we demand $\lambda = 0.22$, equations (23) and (26) become two independent predictions for C_3. *A priori* it is not obvious that the two numbers should even be of the same order of magnitude. When f_θ and k are taken into account, in the context of a particular string compactification, the two predictions will be more than just order of magnitude estimates. If we do *not* require $\lambda = 0.22$, then we have to be able to produce the correct Cabibbo angle from a texture given in terms of powers of the calculated λ.

Furthermore, the charges are integers, and they are constrained by the mixed anomalies. One of the constraints is non-linear. It is not guaranteed

that there will be any solution, much less that it will correspond to realistic masses and mixings.

7.1 Positive Charges for Matter Fields

We begin the search for a detailed model by assuming that all the standard model fields have nonnegative $U(1)_X$ charges. We also assume, following Binétruy and Ramond [4], that because of a possible μ term, the Higgs doublets have zero X charge. We, therefore, form the Yukawa matrices

$$
\mathbf{Y}_u = f_u \begin{pmatrix} \lambda^{(q_{Q1}+q_{u1})/|q_\theta|} & \lambda^{(q_{Q1}+q_{u2})/|q_\theta|} & \lambda^{(q_{Q1}+q_{u3})/|q_\theta|} \\ \lambda^{(q_{Q2}+q_{u1})/|q_\theta|} & \lambda^{(q_{Q2}+q_{u2})/|q_\theta|} & \lambda^{(q_{Q2}+q_{u3})/|q_\theta|} \\ \lambda^{(q_{Q3}+q_{u1})/|q_\theta|} & \lambda^{(q_{Q3}+q_{u2})/|q_\theta|} & \lambda^{(q_{Q3}+q_{u3})/|q_\theta|} \end{pmatrix} \tag{33}
$$

for the up sector, and similarly for the down and lepton sectors. Eq. (33) is an order of magnitude relationship, so that each of the entries will be multiplied by a number of order unity. The factors of order unity will be necessary to introduce CP violation, which we are ignoring in this paper.

We would like to make $|q_\theta|$ the smallest charge unit, i.e. $q_\theta = -1$ (in our convention all X charges are integers). That, however, does not lead to phenomenologically acceptable mass matrices: every row of matrix (33) is equal to $\lambda^{\text{some power}} \times$ (some other row). Similarly, there is only one independent column. The mass matrix, with two zero eigenvalues, does not reproduce the observed mass hierarchy even qualitatively. Although the factors of order unity multiplying the entries in $\mathbf{Y}_u$ will in general move $\det \mathbf{Y}_u$ away from zero, fermion masses and mixings would then very strongly depend on those unknown factors and not on the properties of the model we are trying to investigate. Such a model may be realized in nature, however, we do not know those factors. We will therefore set them to one in this and the following section, and impose conditions to avoid zero determinants. We will find that those conditions are too rigid to produce a realistic example. Since they are not based on physical principles, in section 7.3 we make an arbitrary but limited choice of the "texture factors" to see how much can be achieved by eliminating the artificial constraints.

Without texture factors, any matrix of the form $\mathbf{Y}_{ij} = \lambda^{a_i+b_j}$ will have rank one. A possible solution is to use matrices with texture zeros: $(\mathbf{Y}_u)_{ij} = \lambda^{(q_{Qi}+q_{uj})/|q_\theta|}$ when $(q_{Qi}+q_{uj})/|q_\theta|$ is a nonnegative integer, zero otherwise. If

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the sum of the charges $q_{Qi} + q_{uj}$ is not a multiple of $|q_\theta|$, the corresponding term in the Lagrangian is forbidden by charge conservation. If it is negative, it could only be matched by a power of θ^*. This is impossible in supersymmetric theories because the superpotential must be holomorphic in θ.

With positive charges for the standard model fields, $q_\theta = -1$ does not allow texture zeros. We find that $q_\theta = -2$ is also not enough. The pattern of the texture zeros depends only on the remainder from the division of $q_{Qi} + q_{uj}$ by $|q_\theta|$. Of the three numbers q_{u1}, q_{u2}, q_{u3}, either at least two will be even or at least two will be odd. At least two columns of $\mathbf{Y}_u$, having the same pattern of texture zeros, will be proportional. The result is a matrix of rank at most two.

In order to obtain a non-singular matrix we must have all q_{Qi} different (mod $|q_\theta|$) and all q_{ui} different (mod $|q_\theta|$). That requires $|q_\theta| \geq 3$. In addition, one can see that no row and no column can have more than one nonzero element. The only non-singular mass matrices we can get in this model are rather sparse: they have six texture zeros. Even without the assumption of left-right symmetry [3] there are few possibilities:

$$
\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}, \qquad
\begin{pmatrix} 0 & c & 0 \\ b & 0 & 0 \\ 0 & 0 & a \end{pmatrix}, \qquad
\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & c & 0 \end{pmatrix}, \qquad (34)
$$

$$
\begin{pmatrix} 0 & 0 & b \\ 0 & a & 0 \\ c & 0 & 0 \end{pmatrix}, \qquad
\begin{pmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{pmatrix}, \qquad
\begin{pmatrix} 0 & 0 & c \\ a & 0 & 0 \\ 0 & b & 0 \end{pmatrix}.
$$

In addition, all the above textures can be reduced to a diagonal form by a permutation of columns only. Permutations of columns of a Yukawa mass matrix can be written as a multiplication on the right by a unitary permutation matrix, and it is easy to show that they do not change the resulting masses or mixings. (Permutations of rows, which correspond to multiplication on the left, will not change masses or mixings as long as they are done simultaneously in the up and down sector.) We conclude that the masses and mixings will be the same as if all mass matrices were diagonal, i.e. there will be no flavor mixing.

Can we make things better by giving up some of the assumptions? In the following section we will let all particles have charges of either sign. Here we only note that if we allow the Higgs doublet to have a positive charge (at the

expense of the μ term), we are merely shifting the charges of the up, down or lepton sector without changing any of the conclusions. If the Higgs charge is a multiple of $|q_\theta|$, the textures do not change. Otherwise, the texture zeros will change their positions, but we will still have exactly one nonzero entry in each row and each column.

It should be stressed again that our failure to find a workable example of a model with only positive charges for the standard model fields does not in any way rule out such models. Our conclusion here is that we cannot obtain an acceptable mass matrix to study unless we know the exact factors of order unity in front of the powers of λ in (33). It would be interesting to come back to this exercise when we can make an informed choice of those factors.

7.2   Allowing Negative Charges

We now turn to the analysis of the model in which the matter fields are allowed to have negative as well as positive charges. That will be a source of texture zeros, and give us more flexibility in constructing the mass matrices.

We need to find the conditions necessary to obtain a non-singular matrix, and start by reordering the quarks and leptons so that $q_{Q1} \geq q_{Q2} \geq q_{Q3}$, $q_{u1} \geq q_{u2} \geq q_{u3}$, etc. This is the same as permuting the rows and columns of the mass matrix; it can only change the determinant of $\mathbf{Y}$ by a sign, and will not change the masses or mixings. Now in eq. (33) we put a texture zero wherever $q_{Qi} + q_{uj} < 0$. Keeping in mind that no two columns and no two rows can have the same pattern of texture zeros, we are left with

$$\mathbf{Y}_u = f_u \begin{pmatrix} \lambda^{q_{Q1}+q_{u1}} & \lambda^{q_{Q1}+q_{u2}} & \lambda^{q_{Q1}+q_{u3}} \\ \lambda^{q_{Q2}+q_{u1}} & \lambda^{q_{Q2}+q_{u2}} & 0 \\ \lambda^{q_{Q3}+q_{u1}} & 0 & 0 \end{pmatrix}. \tag{35}$$

We are assuming $q_\theta = -1$ in this section. The texture (35) is rich enough to ask whether it is possible to obtain realistic fermion masses and mixings.

To answer this question, we have done a computerized search by trying out all possible charge assignments for the quarks and leptons (in a range from -10 to 10), imposing the anomaly constraints, calculating the fermion masses and aiming to be as close to the ratios (1) as possible. (We were limited in how close we could get by the relations (23) and (26) between the determinants of the quark mass matrices and the hierarchy parameter λ.) For

those charge sets that produced the best fermion masses, we then computed the CKM matrix. While none of the results reproduce experimental data, we have found many that were not unreasonable. Below we give an example. For $g_s = 0.7$ and the following X charges,

i	q_{Qi}	q_{ui}	q_{di}	q_{Li}	q_{ei}
1	9	9	7	1	10
2	7	2	-4	-7	9
3	-4	-9	-9	-10	3

we have $C_3 = 10$ and $\lambda = 0.30$, which results in the fermion mass ratios

$$\frac{m_u}{m_t} = 1.8 \times 10^{-5} , \qquad \frac{m_d}{m_b} = 2.6 \times 10^{-2} , \qquad \frac{m_e}{m_\tau} = 7.9 \times 10^{-3} ,$$

$$\frac{m_c}{m_t} = 2.3 \times 10^{-2} , \qquad \frac{m_s}{m_b} = 2.6 \times 10^{-2} , \qquad \frac{m_\mu}{m_\tau} = 8.9 \times 10^{-2} ,$$

$$\frac{m_t}{f_u} = 1.0 , \qquad \frac{m_b}{f_d} = 1.0 , \qquad \frac{m_\tau}{f_d} = 1.0 ,$$

and the CKM matrix

$$\mathbf{V} = \begin{pmatrix} 0.96 & 0.27 & 6.2 \times 10^{-5} \\ -0.27 & 0.96 & 2.0 \times 10^{-10} \\ -6.0 \times 10^{-5} & -1.6 \times 10^{-5} & 1.0 \end{pmatrix} . \tag{36}$$

The 90% confidence experimental limits on the magnitude of the CKM matrix elements [30], renormalized to the GUT scale [2], are

$$\begin{pmatrix} 0.9747 \text{ to } 0.9759 & 0.218 \text{ to } 0.224 & 0.001 \text{ to } 0.003 \\ 0.218 \text{ to } 0.224 & 0.9738 \text{ to } 0.9752 & 0.021 \text{ to } 0.032 \\ 0.002 \text{ to } 0.010 & 0.020 \text{ to } 0.032 & 0.9995 \text{ to } 0.9998 \end{pmatrix} . \tag{37}$$

One obvious defect of the above example is that λ and the Cabibbo angle $\mathbf{V}_{12}$ are too big, as discussed in section 6. Another is the degeneracy of the s and d quarks. Again, this is a generic feature of the examples based on the texture (35). The reason is that the dominant terms in (35) are on the antidiagonal; the terms in the upper left triangle are orders of magnitude smaller. Therefore, (35) can be thought of as an antidiagonal matrix with a very small perturbation. Since a permutation of columns can cast it in an

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almost diagonal form, such a matrix will not lead to any appreciable flavor mixing *unless* two of the eigenvalues of $\mathbf{Y}\mathbf{Y}^\dagger$ are degenerate. In such a case the choice of the basis in the eigenspace is arbitrary, and it is easy to obtain large mixing angles. We were looking for $\mathbf{V}_{12} \simeq 0.22$, so it is understandable for all the examples to have degenerate quark masses.

The last thing we noticed is that the "small" entries (1,3), (3,1), (2,3), (3,2) of the CKM matrix (36) are much smaller than what we know from the experiment (even taking renormalization into account). The small mixing in the heavy flavor sector can be understood by noting that the heavy quarks are not degenerate in mass, and the magnitude of the mixing in this case is determined by the magnitude of the off-diagonal perturbation.

7.3 Allowing a Texture Factor

Until now, we have been trying to keep all the factors of order unity in the Yukawa mass matrices equal to one. We tried to avoid the problem of singular mass matrices by restricting our search to matrices with enough texture zeros to be nonsingular. In this section, we want to make the matrices non-singular by introducing coefficients different from unity.

To avoid introducing 27 free parameters, we make a rather arbitrary choice of the coefficients: we introduce one parameter, the "texture factor" (TF), which will multiply the (2,3), (3,2) and (3,3) elements of $\mathbf{Y}_u$ and $\mathbf{Y}_e$. The corresponding elements of $\mathbf{Y}_d$ are divided by TF. That is the minimal intervention needed to make the determinants nonzero in most cases. We chose to modify the entries in the heavy quark sector so that the predicted Cabibbo angle would not depend strongly on TF. We decided to divide, rather than multiply, in $\mathbf{Y}_d$, to make the determinant relations such as (25) minimally sensitive to TF. Finally, we considered only nine discrete values of TF: -1, ± 2, $\pm\sqrt{2}$, $\pm 1/2$, $\pm 1/\sqrt{2}$. With negative as well as positive charges allowed, but without requiring any particular pattern of texture zeros, we were able to find better examples than before. With $g_s = 0.7$ and the X charges

i	q_{Qi}	q_{ui}	q_{di}	q_{Li}	q_{ei}
1	5	3	2	6	5
2	4	2	-1	-2	4
3	-2	1	-1	-5	-1

we have $C_3 = 10$ and $\lambda = 0.30$. With the texture factor TF $= +2$, we find the fermion mass ratios

$$\frac{m_u}{m_t} = 7.5 \times 10^{-6} \,, \qquad \frac{m_d}{m_b} = 3.3 \times 10^{-3} \,, \qquad \frac{m_e}{m_\tau} = 2.3 \times 10^{-3} \,,$$

$$\frac{m_c}{m_t} = 2.3 \times 10^{-3} \,, \qquad \frac{m_s}{m_b} = 3.1 \times 10^{-2} \,, \qquad \frac{m_\mu}{m_\tau} = 8.7 \times 10^{-2} \,,$$

$$\frac{m_t}{f_u} = 2.0 \,, \qquad \frac{m_b}{f_d} = 1.0 \,, \qquad \frac{m_\tau}{f_d} = 1.0 \,,$$

and the CKM matrix

$$\mathbf{V} = \begin{pmatrix} 0.98 & 0.20 & 5.0 \times 10^{-5} \\ -0.20 & 0.98 & 3.5 \times 10^{-4} \\ 2.1 \times 10^{-5} & -3.6 \times 10^{-4} & 1.0 \end{pmatrix} . \tag{38}$$

Although the small elements of the CKM matrix are still about an order of magnitude too small, the above example has no other obvious defects. It reproduces the mass ratios (1) and the Cabibbo angle fairly well, the bottom quark and the tau have equal masses at the unification scale, and the top quark is the heaviest. An even better fit (including the small elements of the CKM matrix) can be obtained for Kac-Moody level $k_{GUT} = 2$, thanks to a much better agreement between (27) and (23). This shows that, finally, it is possible to obtain realistic masses and mixings as a result of an anomalous $U(1)_X$ family symmetry.

8 Conclusions

In this paper, we have examined the idea that the hierarchy parameter in Froggatt-Nielsen type models is related to a spontaneously broken anomalous $U(1)_X$ gauge symmetry left over from string compactification. Given the details of a string compactification, the condition of preserving supersymmetry (7) predicts the value of λ, hence the complete mass matrices, fermion masses and mixings.

Without a complete string model at hand, we looked for model-independent features. We found a strong constraint from anomaly cancellation (17) that gives λ in terms of the quark X charges only. There is another constraint on those charges (27) from the known value of the product of all quark masses.

Assuming $\lambda = 0.22$, the two independent predictions (23) and (28) agree as an order of magnitude relation. Their agreement in a definite model depends on f_θ, the coefficient that enters $\lambda = f_\theta \langle \theta \rangle / M$, and on the Kac-Moody level of the gauge group, k_{GUT}.

We also wanted to verify that it is possible to obtain realistic masses and mixings in this framework. We examine integer charge assignments satisfying the anomaly cancellation constraints, and, with the λ determined by those charges, find some promising examples. It would be very interesting to do the same with a definite string model. The values for f_θ, k_{GUT} and the order unity texture factors in mass matrices would then be specified, leaving no free parameters. The method presented in this paper gives us a powerful tool to narrow down the set of possible string compactifications. We think that there will be only a small number (if any) of models compatible with the idea of predicting λ from $U(1)_X$ charges.

One desirable feature of the CKM matrix we have not addressed in this paper is CP violation. It is possible to introduce CP violation by assuming that the order unity texture factors are complex [1], but that generically leads to large CP violation. Another way is to use a model with two $U(1)_X$ breaking fields, θ_1 and θ_2. The phase of the VEV of a single complex θ field can always be rotated away by a gauge transformation, but for two complex fields there is a gauge-invariant phase difference, so that in general we can make only one of them (say $\langle \theta_1 \rangle$) real.

The powers of $\langle \theta_1 \rangle$ and $\langle \theta_2 \rangle$ will give imaginary parts to the mass matrix elements. If $\langle \theta_2 \rangle \ll \langle \theta_1 \rangle$, then the imaginary parts will be necessarily small, leading to naturally small CP violation. Work on the details of the two-theta model is in progress.

Acknowledgements

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